Technical Journal, University of Engineering and Technology (UET) Taxila, Pakistan Vol. 22 No. 3-2017
ISSN:1813-1786 (Print) 2313-7770 (Online)

Enhanced Performance of Two Phase PMSM and Univariate Non-Stationary Growth Models Through Statistically Linearized Kalman Filter

N. Khan[1], Atta Ullah[2], N. Khan[3], W. Khan[4], K. Akhtar[5]

[1,2,3,4,5]Electrical Engineering Department, University of Engineering and Technology Peshawar, KPK, Pakistan
[5]nkhan@uetpeshawar.edu.pk

Abstract-Traditional schemes of non-linear estimation includes extended Kalman filter (EKF). However due to several shortcomings caused by Jacobian linearization the usage of EKF is problematic. To avoid the problems linked with Jacobian linearization, this paper presents Kalman filtering technique based on statistically linearization. The derivation of this nonlinear estimation scheme has been achieved by steps similar to standard Kalman filter (KF) techniques. The system is linearized through statistical linearization rather than Taylor series. This statistically linearization is implemented to obtain the state of two important models, namely two phase permanent magnet synchronous motor (PMSM) and univariate non stationary growth model. It has been shown that the schemes has generated improved performance than EKF. Various performance indeces have been shown for performance comparison. Results obtained through two estimation techniques are compared with the actual state values. The results obtained through proposed scheme are significantly improved compared to the results obtained for existing schemes. In consequence, the error linked with proposed estimation techniques has been greatly minimized through the use of statistically linearized KF.

Keywords-Nonlinear Filter, Extended Kalman Filter, Statistical Linearization, Global Approximation, Jacobian Matrix, Taylor Series.

I. INTRODUCTION

The process of filtering and estimation has remained one of the most investigated phenomenon in numerous engineering applications. For example, a standard Gaussian noise may corrupt the health and quality of radio communication signals in various perspectives. An efficient and robust algorithm would be the one that could retain information while discarding the unwanted signal. A notable example is UPS (Uninterruptible Power Supply) which are designed to rectify line-voltage for smoothing purposes. These removed fluctuations might hinder the performance of equipment and affect the life span of connected devices. In this connection. Kalman filter is a well-known, abundantly employed and an established optimal estimator for linear system. Kalman filter has the beauty that it can handle both transition and measurement system noises [i]. The propagation of Gaussianity via system dynamics is the central operational point of KF.

The development of Kalman filter has modernized the field of estimation, and has a massive impact on the design and development of accurate navigation systems [i]. It has been used in almost all modern control and communication systems, both military and space technology such as in inertial guidance systems in aircraft [ii], missile autopilots, submarines, phased-array radars to track missiles, the Global Positioning System(GPS) [iii], the space shuttle, rockets [iv] and in separating the speech signals under additive white Gaussian noise channel [v].

However, in majority of applications, the system under observation is nonlinear. Hence filtering schemes were immediately modified to cope the situations including nonlinearity. In modern era, nonlinear filtering and estimation have been a subject of active research such as signal processing, navigation, control, target tracking, neural network training and majority of electrical/electromechanical systems [vi-viii].

For handling nonlinear functions nonlinear estimation tools including the extended Kalman filter (EKF) [ix-x] and unscented Kalman filter (UKF) [xi] are widely used. Generally speaking, in estimation theory EKF may be called the nonlinear adaptation of the linear KF as it linearizes a nonlinear function of the system model around the current state estimate. In this nonlinear estimation scheme, the predicted state is approximated through a GRV that propagate analytically through first order Taylor series. Since EKF employs a posterior mean and covariance entities so it may lead to sub-optimal results. This may cause divergence of filter ultimately. Proposed work in this article that is statistically linearized Kalman filter (SLKF) deal with the mentioned problem resulting optimal estimation. Another dilemma associated with EKF is that, numerical Jacobians are required in the absence of analytical Jacobians. As in linearization of the nonlinear system EKF uses first order Taylor series so it encounters issues with both accuracy and stability.In contrast to this SLKF uses statistical linearization which avoids majority of problems related to first order Taylor series. In addition to this EKF also needs complex evaluation of Jacobian matrices of the nonlinear system dynamics and measurement functions which are not required in SLKF. Another drawback of EKF is that it fails in some cases, where Jacobian matrices of measurement functions become zeros [xii-xiii], while SLKF has the ability to estimate in such cases.

UKF is a derivative free alternative to EKF and is proposed by Julier and Uhlman [xiv-xv], addresses these problems by using deterministic sampling approach. The approximation process of state. The approximation process of state distribution is repeated through GRV which is depicted by a minimal set of carefully selected points called sigma points [xvi]. Sigma points convey the actual mean and covariance of the associated GRV. These points when proliferated through any nonlinear system, conquers the subsequent mean and covariance exactly to 3rd order (Taylor series expansion) for any nonlinear system.

On the other hand, EKF, only provides 1st order Taylor series accuracy. In this regards, UKF has superior performance in comparison to linear Kalman filter and EKF [xiii], as it does not need of evaluation of Jacobian matrices [xvii]. The proposed work completely avoids Taylor series and hence there is no complexity of evaluation of Jacobian matrices

The EKF linearizes a nonlinear function in the vicinity of current state estimate and consequently the state theoretically preserves its Gaussianity in course of the whole time interval. On the other hand, UKF approximates expectation and covariance matrix of the nonlinearly transformed state which creates non Gaussian distribution of the state. Since the Gaussianity can be manipulated in designing the optimal filter so EKF is effective. However, UKF provide improved state estimation than EKF using unscented transformation techniques. Approaches where state Gaussianity is needed UKF are not applicable. For this reason, statistical linearization (SL) technique is proposed which is expected to be superior in managing the nonlinear functions over the Taylor series truncation approach of the EKF.

The paper is presented as follow: Section II declares the problem statement and some shortcomings of EKF and UKF. This section also include the proposed solution and familiarizes some necessary work about statistical linearization and statistically linearized Kalman filter (SLKF). In section IV two case studiesnamely two PMSM and univariate non-stationary growth model are implemented and states are estimated using EKF and SLKF. In section V a conclusion is drawn and the performance of the two models using EKF and SLKF are discussed.

II. Problem Formulation

In this paper, the estimation problem associated with Extended Kalman filter, (where Taylor series is employed for linearization of the non-linear function)has been address. This issue is solved using the statistical approach of linearization which is equally applicable to both linear systems and as well as nonlinear systems. The EKF is often considered to be an excessively complex algorithm, which is difficult to implement as it requires huge computational efforts. Unusually computational complexity of UKF' and EKF is of the same order [xiv]. In EKF, 1st order Taylor series is used for linearization of the nonlinear functions, and has some serious limitations [xii-xiii, xviii] as listed below.

- EKF uses 1st order Taylor series for linearization of the nonlinear system so it encounters issues with both accuracy and stability.
- Secondly, the EKF gives poor estimation in case of highly nonlinear systems.
- Third limitation is to calculate complicated Jacobian matrices of the nonlinear system and measurement functions.

It fails in cases where Jacobian matrices of measurement functions are not of full rank.

In order to avoid these limitations of EKF and to obtain a better state estimation of a system, in this paper an improved nonlinear Kalman filter called equivalent linearization Kalman filter (EqKF) is proposed. Since the approach adopted in this proposed scheme is based on statistical linearization, it can also be called Statistical-Linearized KF or SLKF [xiv]. The EqKF or SLKF is presented as an alternative to EKF and UKF. The statistical linearization approach was developed by Kazakov, et al [xx], Caughey, et al [xxi] and others have improved and analyzed various nonlinear stochastic dynamic systems.

This research paper attempts to implement the SLKF for the case study of two phase PMSM and to avoid the limitations associated with EKF. In order to show that the proposed SLKF algorithm covers the limitations of highly nonlinear system, another case study -- the univariant non-stationary growth model, a highly nonlinear system has been tested to compare the performances.

Linearization procedure is discussed in the next section.

A. Statistical linearization for Gaussian approximation

There are two ways, where a nonlinear system can be linearized: statistical linearization and unscented transformation. The later one is adopted in Unscented Kalman filtering. The unscented linearization varies from system to system as it is not a series based method and is a complicated technique [xxii]. For such reasons, in this paper we are considering statistical linearization.

In this section, a brief mathematical review of statistical (or equivalent) linearization for nonlinear function is given. Let $x \in R^n$ be a Gaussian random vector with mean $m_x \in R^n$ and covariance matrix $P \in R^{n\times n}$. Let a nonlinear function be $y = h(x) : R^n \to R^p$. We derive a linear unbiased minimum variance estimate *y-hat* of *y* as shown in Fig. 1:

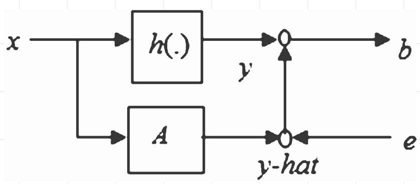

Fig. 1.

Let $\hat{y} = b + Ax$, where $A \in R^{p\times n}$, $b \in R^p$ then the un biasedness of $\hat{y}$ implies that $E[y - Ax - b] = 0$, so that $b = m_y - Am_x$, where $m_y = E[y] = E[(x)]$.

Let $x \sim N(m,p)$ and $y = h(x)$, then linear approximation is: $h(x) \approx b + A\delta +$, where $\delta x = x - m$ and m is mean value of x.

The basic purpose of this method of linearization is to replace the nonlinear components in a model by linear forms where the coefficients of linearization can be set up centered on a particularized criterion of linearization. The coefficients of linearization are chosen in such a way that the error in the approximation method becomes a minimum. The mean squared error can be minimized as follows without using Taylor series [xxii]:

$$MSE(b, A) = E[h(x)] - E[(h(x) - b - A\delta x)^T (h(x) - b - A\delta x)] \quad (1)$$

Expounding the MSE expression results in:

$$MSE(b, A) = E[h^T(x)h(x) - 2h^T(x)b - 2h^T(x)A\delta x + b^T b - 2b^T A\delta x + \delta x^T A\delta x \quad (2)$$

The values of A and b are found in such a manner that minimize the mean square error.

$$\frac{\partial MSE(b, A)}{\partial b} = -2E[h(x)] + 2b \quad (3)$$

$$\frac{\partial MSE(b, A)}{\partial A} = -2E[h(x)\delta x^T] + 2AP \quad (4)$$

Equating the derivatives in equations (3) and (4) equal to zero yields:

$$b = E[h(x)] \quad (5)$$

$$A = E[h(x)\delta x^T]P^{-1} \quad (6)$$

The values in Eq. 5 and 6 will yield minimum possible error. Now if the covariance of x is P and A is a known matrix then covariance of $h(x)$ is given by:

$$Cov[h(x)] \approx APA^t = E[h(x)\delta x^T]P^{-1} E[h(x)\delta x^T]^T \quad (7)$$

Equation (7) can be obtained simply by substitution of Eq. (6).

In case of joint distribution of x and $y = h(x) + q$ where $x \sim N(m,p)$ and $q \sim N(0,Q)$, the statistically linearized Gaussian approximation is given as:

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim N\left(\begin{bmatrix} m \\ \mu_s \end{bmatrix}, \begin{bmatrix} P & C_S \\ C_S^T & S_S \end{bmatrix} \right) \quad (8)$$

where $\mu_s = E[h(x)]$ (9)

and C_s is given by:

$$C_s = A^t = E[h(x)\delta x^T]^T \quad (10)$$

which is the mean of *h(x)*.

$$S_s = Cov[h(x)] + Q = E[h(x)\delta x^T]P^{-1}E[h(x)\delta x^T]^T + Q \quad (11)$$

In Eq. (11) Q is the covariance matrix.

Now it will be shown that the linearization used in EKF is local method while the linearization used in SLKF is global method. There are two schemes for linear approximation of a nonlinear function $[h(x)]$, in which one is local approximation, i.e.

$$h(x) \cong h(m) + \left[\frac{\partial h(x)}{\partial x}\right]_{x=m} (x - m) \quad (12)$$

and the other is global approximation:

$$h(x) \cong E\{h(x)\} + E\left\{\frac{\partial h(x)}{\partial x}\right\}(x - m) \quad (13a)$$

$$= E\{g(x)\} + \frac{\partial}{\partial m} E\{g(x)\}(x - m) \quad (13b)$$

It is well recognized that the EKF is derived by means of local approximation of Eq. (12), whereas the SLKF is derived by mean of global approximation of Eq. (13).

B. Statistically Linearized Kalman Filter (SLKF)

In this paper, it is shown that employing SLKF gives better estimation compare to the existing EKF scheme. A numerical model of a two-phase permanent magnet synchronous motor (PMSM) is considered for evaluating the performance of both schemes. First order linearization of EKF is a local method as it is derived using a local approximation while statistical linearization of SLKF is a global method because it is derived by using a global approximation. Similar to standard KF, the SLKF works in two steps given below [xxv]:

Prediction expectations with respect to $x_{K-1} \sim N(m_{K-1}, P_{K-1})$ is given by:

$$\overline{m}_k = E[h(x_{k-1})] \quad (14)$$

where 'm' is mean value of vector and 'P' is error covariance. The covariance matrix is based on prediction as follow:

$$\overline{P}_k = E[h(x_{k-1})\delta x_{k-1}]P_{k-1}^{-1}E[h(x_{k-1})]\delta x_{k-1}^T]^T + Q_{k-1} \quad (15)$$

where Q is the covariance matrix.

Update (expectations w.r.t. $x_k \sim N(\overline{m}_k, \overline{p}_k)$

$$v_k = y_k - E[h(x_k)] \quad (16)$$

In equation (16) v_k is residual and y_k is the output.

$$S_k = E[h(x_k)\delta x_k^T](\overline{P}_k)^{-1}E[h(x_k)\delta x_k^T]^T + R_k \quad (17)$$

where S_k is the residual covariance used in updated step as shown below:

$$K_k = E[h(x_k)\delta x_k^T]^T S_k^{-1} \quad (18)$$

$$m_k = \overline{m}_k + K_k v_k \quad (19)$$

$$P_k = \overline{P}_k - K_k S_k K_k^T \quad (20)$$

The SLKF scheme is claimed to avoid the limitations associated with EKF by approximating the Jacobian matrix of the system in a broader region centered at the state of the system. This type of methodology also has the advantage that it does not need the differentiability or continuity of the system and observation dynamics models. In view of the fact, it is not indispensable to calculate Jacobian matrices. These methods can offer benefits in terms of computational competence. However, the complete and honest analysis reveals a drawback that SLKF necessitates the nonlinear functions to be given in the closed form. In order to watch the performance of proposed SLKF, two case studies are tested in the subsequent section.

III. Numerical Model

In this research work, two case studies are considered for implementation of EKF and SLKF algorithms. The first case study is a nonlinear system of two phase PMSM and the other case study is a highly nonlinear system of univariant non-stationary growth model. From the first case study, it has been revealed that under same conditions for the a system model, the SLKF algorithm gives better performance in various indexes as compared to EKF algorithm. Also, the proposed SLKF avoids the other limitations faced by EKF. A comparatively efficient estimation (in term of absolute error) can be achieved using proposed SLKF. In the second case study, it will be shown that the SLKF also gives better performance for a highly nonlinear system compared to EKF.

A. Two Phase PMSM.

State estimation of two-phase PMSM system is of greater concern because for regulation of control mechanism, knowledge of the states is of prior importance. It is supposed that currents of the motor windings (primary and secondary) are measured. The basic purpose is to implement EKF and SLKF for the stated case study. The electromechanical system of two-phase PMSM is given by the following equations [ix, xxi, xxiii].

$$I_a = \frac{-R}{L} I_a + \frac{w\lambda}{L}\sin\theta + \frac{u_a + \Delta u_a}{L} \quad (21)$$

$$I_b = \frac{-R}{L} I_b + \frac{w\lambda}{L}\cos\theta + \frac{u_b + \Delta u_b}{L} \quad (22)$$

$$\dot{\omega} = \frac{-3\lambda}{2j} I_a \sin\theta + \frac{3\lambda}{2j} I_b \cos\theta - \frac{F\omega}{j} + \Delta\alpha \quad (23)$$

$$\dot{\theta} = \omega \quad (24)$$

$$y = \begin{bmatrix} I_a \\ I_b \end{bmatrix} + \begin{bmatrix} v_a \\ v_b \end{bmatrix} \quad (25)$$

- The variables in the above equations are defined as give below:
- I_a is current in primary winding and I_b is current in secondary winding of the motor.
- θ is the angular position and ω is the angular velocity of the rotor.
- L represents inductance and R represents resistance of the motor windings.
- λ represents flux linkage constant.
- F symbolizes the coefficient of viscous friction which is acting on the motor shaft and the load attach to it.
- J represents moment of inertia of the shaft of the motor and its load
- μ_a and μ_b symbolizes the applied voltages across the two windings of the motor.
- $\Delta\mu_a$ is the noise terminus in μ_a and $\Delta\mu_b$ is the
- noise terminus in μ_b
- Δω is a noise terminus because of uncertainty in the torque of the load
- y is the measurement.

Assume that measurements i.e. motor's windings currents are achieved through sense resistors only. In such case, measurements are degraded by measurement noise v_a and v_b, caused by quantities like electrical noise, uncertainty in sense resistance, and errors as a result of quantization in microcontroller.

The four states linked with the above system are

$$x = \begin{bmatrix} I_a \\ I_b \\ \omega \\ \theta \end{bmatrix} = \begin{bmatrix} x_{k1} \\ x_{k2} \\ x_{k3} \\ x_{k4} \end{bmatrix}$$

Initially, EKF is employed to estimate the states. Thereafter, for performance evaluation SLKF is also employed for the same case study. Simulation results are shown in the following Figures

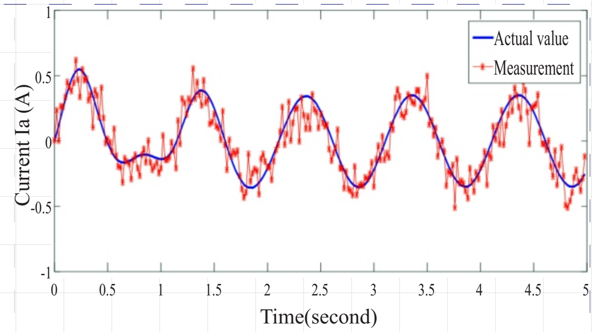

Fig. 2. Actual and noisy measurement current value for first winding of the two-phase PMSM.

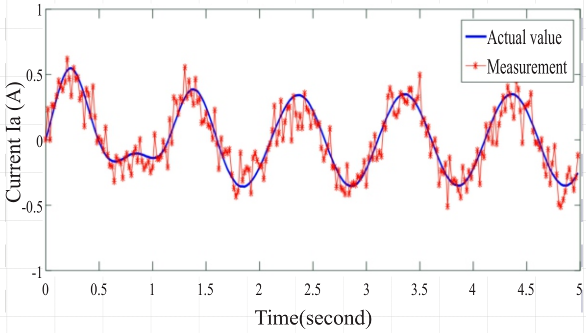

Fig. 3. Actual and noisy measurement current value for second winding of the two-phase PMSM (Measurement of winding currents are prevailed once per millisecond).

Figures 2 and 3 represent the actual and noisy data of the two winding currents wherein the continuous line (in blue color) shows the actual state and dot-marked (red) line represents the measured data. As shown in the Figures the measured data is interrupted by noise. Although the measurements comprise only winding currents but we will use EKF and SLKF for estimation of other states as well (i.e. position and velocity of the rotor) associated with the case study. These states are estimated as shown in the subsequent results.

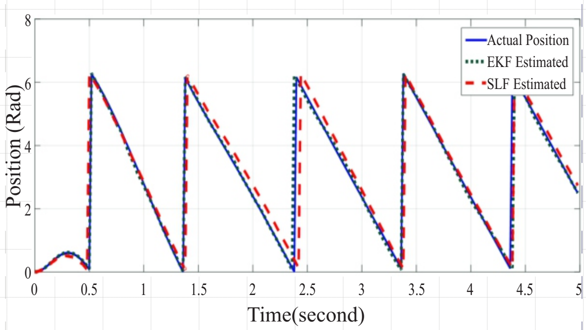

Fig. 4. EKF and SLKF Estimated position of the rotor of a two-phase PMS

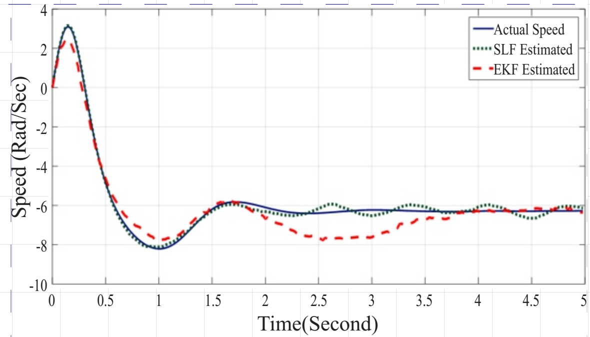

Fig. 5. EKF and SLKF Estimated position of the rotor of a two-phase PMSM

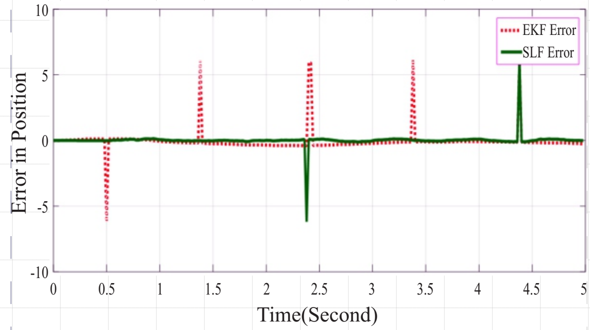

Fig. 6-A. Error analysis in the position state

Figures 4 and 5 show the estimation results for EKF and SLKF for the position and speed of the two-phase motor. These Figures show the actual states and estimated states generated by both EKF and SLKF. In the figures continuous line shows the actual states, dashed lines represents estimation by EKF and dotted line shows estimation by SLKF. The estimation results of the two schemes (dashed and dotted lines) are compared with the actual value (continuous line). It can be seen that the proposed model based on SLKF tracks the actual state much better than that of EKF algorithm.

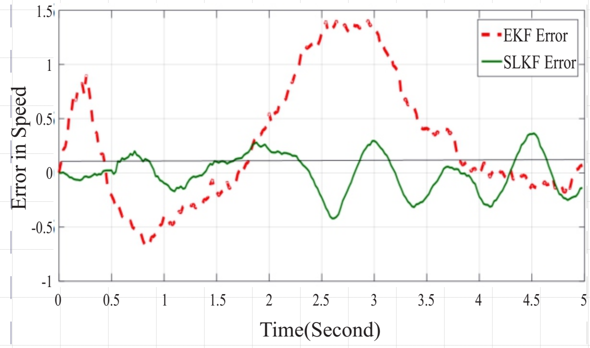

Fig. 6-B. EKF and SLKF Error results for two-phase PMSM

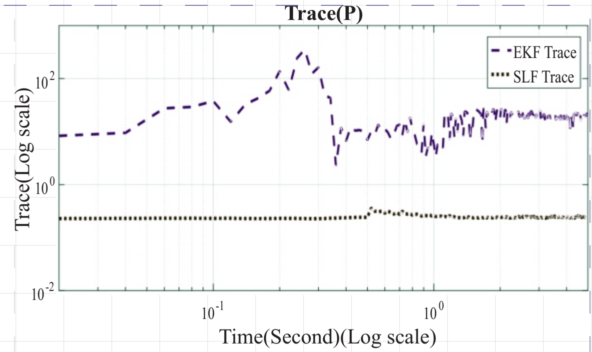

Fig. 7. EKF and SLKF Trace results

Another view of the performances of the proposed filter and existing EKF can be seen in Figures 6-A and 6-B. This figure reveals the corresponding errors signal resulted by the two algorithms under-consideration. Frequent spikes and large error can be observed in the existing EKF algorithm. However, in the results obtained for SLF, the error is quite improved by reducing its values.

Similarly, Figure 7 which is actually the trace or error covariance shows that the sum of square of errors of SLKF is much smaller compared to EKF, in estimating the states of a two phase PMSM.From Figures 6-A, 6-B and 7, the improvement made through proposed model is 12.34% approximately compared to the existing EKF scheme.

The results shown in this section justifies the enhanced performance of proposed SLKF over the exiting EKF. The system discussed in this sectionis not so highly nonlinear system. In order to testify the performance of the proposed scheme for a highly nonlinear system, another case study, which is univariate non-stationary growth model is considered.

A. Univariate Non-Stationary Growth Mode.

In order to further evaluate the scope of this work, a highly nonlinear system has been taken into consideration. Non-stationary model can occur in many ways such as non-constant means, non-constant variances and seasonal models, etc. Seasonal series can be described by a strong serial correlation at the seasonal retardation. Univariate non-stationary growth model has been used extensively in literature to authenticate the performance of nonlinear filters because of its excessive nonlinearity and popularity in econometrics [xxv]. Nonlinear process and measurement equations of this case study can be formulated as [xxiv]

$$= 0.5x_{k-1} + 10\frac{x_{k-1}}{1+x_{k-1}^{2}} + 8\cos(1.2k) + w_{k-1} \quad (27)$$

$$z_k = \frac{xk^3}{20} + v_k \quad (28)$$

Where $x_0 = 0.1$ is the initial actual state, $\hat{x}_{0|0} = 0$ is the initial estimated state, $P_{0|0} = 1$ is the gibing initial estimate error variance, W_k is the process noise, and V_k is the measurement noise. Both the noises are assumed to be uncorrelated zero mean Gaussian white process with $Q_k = 10$ and $R_k = 100$. Simulation time is taken to be T = 10(years). The model under consideration is actually seasonal model of artificially generated monthly data. Implementing EKF and SLKF for univariate non-stationary growth model the results obtained are given below:

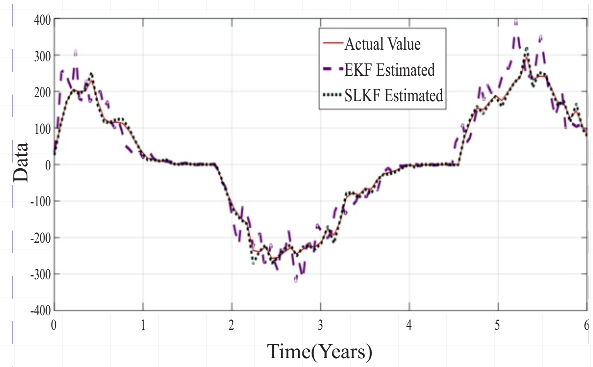

Fig. 8. EKF and SLKF Estimated states for a univariate non-stationary growth model.

Fig. 8 depicts the performance of the two mentioned techniques namely extended Kalman filter (EKF) and statistically linearized Kalman filter (SLKF). Estimation results obtained through both techniques are compared with the actual value as a reference signal. In this Fig. continuous line shows the actual state, dotted line represents the estimated state by SLKF and dashed line shows the estimation by EKF. The result obtained through SLKF scheme is significantly closer to the actual value compared to the results obtained from EKF. Fig. 9 reveals the performance of SLKF and EKF in the index of absolute error signal.

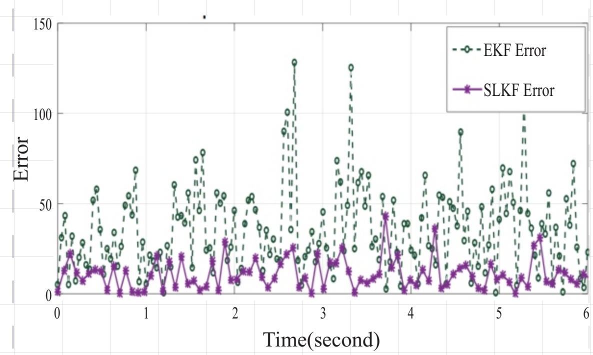

Fig. 9. EKF and SLKF error results for a univariate non-stationary growth model.

It can be seen that the results achieved through SLKF method (Dotted line) have smaller error than the results obtained through EKF technique, which manifest superior performance of SLKF. A Monto-Carlo of 1000-run reveals an estimated error of 1.92% for the proposed scheme while for the EKF 5.43%. It shows that the proposed model has reduced more than three (03) fold error compared to the existing one.

IV. Conclusion

In this paper, implementation of statistically linearized Kalman Filter for two case studies namely two phase permanent magnet synchronous motor and the generalized univariant non-stationary growth model is presented. The core purpose of this paper was to avoid the shortcomings associated with standard extended Kalman filter, especially in dealing highly nonlinear systems. Efforts were made to take maximum feature of the mentioned two schemes of estimation into account. The simulation results have shown an enhanced performance of SLKF compare to EKF. Error in estimation of SLKF is much lesser than EKF and also the Trace for the SLKF is much better than EKF. It is also obvious from simulation results that the SLKF gives better performance for highly nonlinear system than EKF. Finally, it has been brought to a conclusion that instead of using EKF for estimation of a system's states, one can get better performance using SLKF.

References

[i] R. G. Brown and P. Y. C. Hwang, (2012) Introduction to Random Signals and Applied Kalman Filtering. 4th Edition, John Wiley & Sons, Inc., Hoboken

[ii] V. Klein and E.A. Morelli., "Aircraft System Identification. Theory and Practice," AIAA Education Series, 2006.

[iii] M. S. Grewal, L. R. Weill, A. P. Andrews, "Global Positioning Systems, Inertial Navigation, and Integration, Second Edition, " John Wiley & Sons, Inc., Publication, 2007

[iv] M. R. Ananthasayanam, A. K. Anilkumar and P. V. Subba Rao, "New Approach for the Evolution and Expansion of Space Debris Scenario" Journal of Spacecraft and Rockets, Vol. 43, No. 6: pp. 1271-1282, 2006.

[v] Dionelis, N., & Brookes, M. (2017). Modulation-domain speech enhancement using a Kalman filter with a Bayesian update of speech and noise in the log-spectral domain. *2017 Hands-free Speech Communications and Microphone Arrays (HSCMA)*.

[vi] Courses, E.; Surveys, T. (2006)."Sigma-Point Filters: An Overview with Applications to Integrated Navigation and Vision Assisted Control". *IEEE Nonlinear Statistical Signal Processing Workshop*, Cambridge, UK: pp. 201–202.

[vii] Van der Merwe, R. and E. A. Wan (2001), Efficient Derivative-Free Kalman Filters for Online Learning, In European Symposium on Artificial Neural Networks (ESANN), Bruges, Belgium, pp. 205-210.

[viii] MA Jing-bo, YANG Hong-geng. Application of adaptive Kalman filter in power system short-term load forecasting [J]. Power System Technology, 2005, 29(1): 75-79

[ix] D. Simon (2006), "Optimal State Estimation: Kalman, $H\infty$, and Nonlinear Approaches," John Wiley & Sons Inc.

[x] P. D. Groves (2008), Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems. Norwood, MA: Artech House, 2008.

[xi] Peihua, Li and Tianwen Zhang (2004). Unscented Kalman Filter for Visual Curve Tracking. In Proceedings of Statistical Methods in Video Processing, 22(2), pp. 157–164.

[xii] Alghamdi, A. S., Ali, M.N. and Zohdy, M.A. (2015) Robust Non-Coherent Demodulation Scheme for Bluetooth Voice Transmission Using Linear, Extended, and Unscented Kalman Filtering. Journal of Signal and Information Processing, 6, pp 9-27.

[xiii] Xing, Z. and Xia, Y. (2016). Comparison of centralised scaled unscented Kalman filter and extended Kalman filter for Multisensor data fusion architectures. IET Signal Processing.10 (6) 2016 pp. 359 - 365.

[xiv] S. J. Julier and J. K. Uhlmann (1997). A New Extension of the Kalman Filter to Nonlinear Systems. In *Proc. of AeroSense: The 11th Int. Symp. On Aerospace/Defence Sensing, Simulation and Controls,* Vol. 3068 pp. 182—193.

[xv] S. Julier, J. Uhlmann, and H.F. Durrant-Whyte (2000) "A New Method for the Nonlinear Transformation of Means and Covariances in Filters and Estimators". IEEE Trans. Automatic Control, 45,(3), pp. 477–482.

[xvi] S. J. Julier and J. K. Uhlmann (2004). Unscented Filtering and Nonlinear Estimation. Proceedings of the IEEE, 92 (3): pp 401–422.

[xvii] Wan and R. van der Merwe (2001). Chapter 7, The Unscented Kalman Filter. Wiley Publishing.

[xviii] Farina, A., Ristic, B. and Benvenuti, D. (2002). Tracking a ballistic target: comparison of several nonlinear filters. *IEEE Trans. Aerosp. Electron. Syst.*, 38 (3), pp.854-867.

[xix] J. B. Roberts and P. D. Spanos, Random Vibration and Statistical Linearization, Wiley 1990 (Dover edition 2003).

[xx] E. Kazakov (1956), Approximate probabilistic analysis of the accuracy of operation of essentially nonlinear systems, Automation &

Remote Control, 17 (5), pp. 423–450.

[xxi] T. K. Caughey (1963), Equivalent linearization technique, J. Acoustical Soc. America, 35 (11), pp. 1706–1711.

[xxii] Sa̋rkka¨Simo. (2013). *Bayesian filtering and smoothing*. Cambridge: Cambridge University Press.

[xxiii] Dr. A. J. Hamidi, A.A. Olga & Y. N. Ibrahem. (2009). State Estimation of Two-Phase Permenant Magnet Synchromous Motor. Engineering and technology journal, Vol. 27, No. 7, pp. 1435-1443

[xxiv] Clements, M. and Hendry, D. (2001). *Forecasting non-stationary economic time series*. Cambridge: The MIT Press.

[xxv] Laura Mayoral Winter (2012) *Introduction to univariate Nonstationary time series models*,: BGSE.